

2010 Q2.

$$(a)(i) \begin{cases} \vec{V}_A = 4\vec{i} - 3\vec{j} \\ \vec{V}_B = 5\vec{i} - 7\vec{j} \end{cases} \left\{ \begin{aligned} \vec{V}_{BA} &= \vec{V}_B - \vec{V}_A \\ &= (5\vec{i} - 7\vec{j}) - (4\vec{i} - 3\vec{j}) \\ &= (5-4)\vec{i} + (-7+3)\vec{j} \\ &= \vec{i} - 4\vec{j} \end{aligned} \right.$$

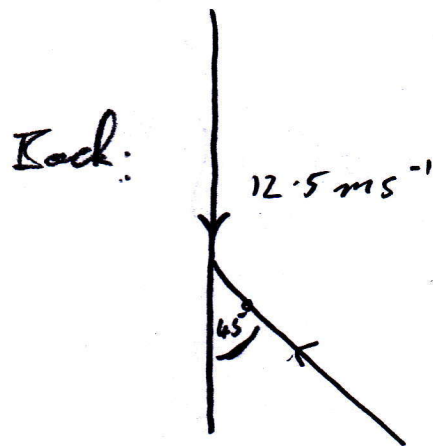
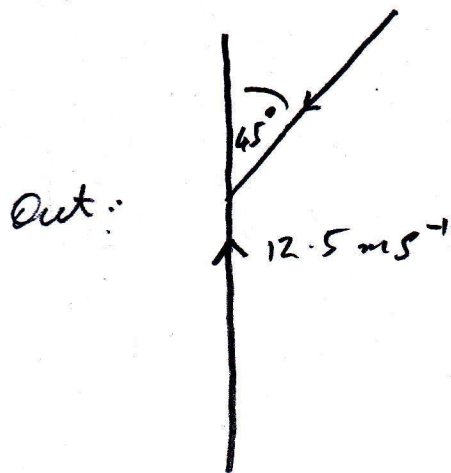
$$\text{Magnitude:} = \sqrt{(1)^2 + (-4)^2} = \sqrt{17} \text{ m s}^{-1}$$

$$\text{Direction:} = E \tan^{-1}(4) S \quad \text{i.e. } \tan \alpha = -\frac{4}{1}$$
$$= E 75.96^\circ S \quad \boxed{\text{slope} = -4}$$

$$(ii) \begin{cases} \vec{V}_A = 6\vec{i} - 14\vec{j} \\ \vec{V}_B = 3\vec{i} - 2\vec{j} \end{cases} \left\{ \begin{aligned} \vec{V}_{BA} &= (3\vec{i} - 2\vec{j}) - (6\vec{i} - 14\vec{j}) \\ &= -3\vec{i} + 12\vec{j} \\ &= -3(\vec{i} - 4\vec{j}) \end{aligned} \right.$$

Since $\vec{V}_{BA} = -k(\vec{V}_A)$ where $(k = \frac{1}{3})$
they are on a collision course.

(b)



Out:

$$\vec{V}_M = 12.5 \vec{j}$$

$$\vec{V}_{WM} = -x \vec{i} - x \vec{j} \quad (\text{since angle is } 45^\circ)$$

$$\Rightarrow \vec{V}_W = \vec{V}_{WM} + \vec{V}_M$$

$$= -x \vec{i} + (12.5 - x) \vec{j} \quad \textcircled{1}$$

Back:

$$\vec{V}_M = -12.5 \vec{j}$$

$$\vec{V}_{WM} = -y \vec{i} + y \vec{j}$$

$$\vec{V}_W = \vec{V}_{WM} + \vec{V}_M$$

$$= -y \vec{i} + (y - 12.5) \vec{j} \quad \textcircled{2}$$

Since $\textcircled{1} = \textcircled{2}$

$$\Rightarrow -x \vec{i} + (12.5 - x) \vec{j} = -y \vec{i} + (y - 12.5) \vec{j}$$

$$\Rightarrow x = y \quad \text{and} \quad 12.5 - x = y - 12.5$$

$$\Rightarrow 12.5 - y = y - 12.5$$

$$\Rightarrow 25 = 2y$$

$$\Rightarrow 12.5 = y$$

$$\Rightarrow \vec{V}_W = -12.5 \vec{i} + 0 \vec{j} \quad \text{with Magnitude } 12.5 \text{ m/s and dir.} = \text{West.}$$