2011 Q10.

(a) $$x^2 \frac{dy}{dx} - xy = 7y$$

$$\Rightarrow \frac{dy}{dx} = \frac{7y + xy}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(7+x)}{x^2}$$

$$\Rightarrow \frac{1}{y}\, dy = \frac{7+x}{x^2}\, dx$$

$$\Rightarrow \int \frac{1}{y}\, dy = \int \frac{7+x}{x^2}\, dx \quad \left[= \int \left(\frac{7}{x^2} + \frac{1}{x} \right) \right]$$

$$\Rightarrow \ln y = -7x^{-1} + \ln x + c$$

$$y = 1 \;,\; x = 1$$

$$\Rightarrow \ln 1 = -7(1)^{-1} + \ln 1 + c$$

$$\Rightarrow 0 = -7 + 0 + c$$

$$\Rightarrow 7 = c$$

$$\Rightarrow \ln y = \ln x - \frac{7}{x} + 7$$

$$x = 2 \Rightarrow \ln y = \ln 2 - \frac{7}{2} + 7$$

$$\Rightarrow \ln y = 4.1931$$

$$\Rightarrow y = 66.23$$

(b) (i) Since we need distance, use $v\frac{dv}{dx}$

$$v\frac{dv}{dx} = -\left(\frac{v^2}{400} + 16\right)$$

$$= -\left(\frac{v^2+6400}{400}\right)$$

$$= -\frac{1}{400}\left(v^2+6400\right)$$

$$\Rightarrow \frac{v}{v^2+6400}\,dv = -\frac{1}{400}\,dx$$

$$\Rightarrow \int_{40}^{0}\frac{v}{v^2+6400}\,dv = -\frac{1}{400}\int_0^x dx$$

$$\Rightarrow \left[\frac{1}{2}\ln\left(v^2+6400\right)\right]_{40}^{0} = -\frac{1}{400}\left[x\right]_0^x$$

$$\Rightarrow \frac{1}{2}\ln 6400 - \frac{1}{2}\ln 8000 = -\frac{x}{400}$$

$$\Rightarrow 4\cdot3820 - 4\cdot4936 = -\frac{x}{400}$$

$$\Rightarrow 44\cdot63 = x$$

(ii) $$\frac{dv}{dt} = -\left(\frac{v^2+6400}{400}\right)$$

$$\Rightarrow \int_{40}^{0}\frac{1}{v^2+6400}\,dv = -\frac{1}{400}\int_0^t dt$$

$$\Rightarrow \left[\frac{1}{80}\tan^{-1}\left(\frac{v}{80}\right)\right]_{40}^{0} = -\frac{1}{400}\left[t\right]_0^t$$

[See tables p26. Note: $\int\frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$]

$$\Rightarrow -\frac{1}{80}\,\mathrm{Tan}^{-1}\left(\frac{40}{80}\right) = -\frac{t}{400}$$

$$\Rightarrow \frac{400}{80}\,\mathrm{Tan}^{-1}\,\frac{1}{2} = t$$

$$\Rightarrow \quad 2.32 \text{ sec} = t$$

[Use Radian mode]

$$\text{Average speed} = \frac{44.63}{2.32}$$

$$= 19.24 \text{ ms}^{-1}$$