

# Solutions to Sample Questions, Higher Level.

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## Question 1

(25 marks)

A survey of 50 Leaving Certificate candidates in 2014, randomly selected in the Dublin region, found that they had a mean mark of 374 in a certain subject. The standard deviation of this sample was 45.

- (a) Find the 95% confidence interval for the mean mark in the subject, in the Dublin region. Interpret this interval.

Solution:

Confidence interval for mean,  $\mu$ :

$$\bar{x} - 1.96 \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96 \frac{s}{\sqrt{n}}$$

$$374 - 1.96 \left( \frac{45}{\sqrt{50}} \right) \leq \mu \leq 374 + 1.96 \left( \frac{45}{\sqrt{50}} \right)$$

$$374 - 12.47 \leq \mu \leq 374 + 12.47$$

$$361.53 \leq \mu \leq 386.47$$

Note: The interpretation is that we have 95% confidence that the true mean is in this interval. This does not mean that we believe there is a 95% chance that the true mean is in it (the idea is that there is some true mean which is either inside the interval, or isn't - no probability involved), only that if we performed this data-collection and generated a confidence interval many times, about 95% of those intervals would capture the true mean [rather subtle point... a lot of statisticians get this wrong].

- (b) The mean mark in the subject for all Leaving Certificate candidates, in 2014, was 385 and the standard deviation was 45. John suggests that the mean mark in the Dublin region is not the same as in the whole country. Test this hypothesis using a 5% level of significance. Clearly state your null hypothesis, your alternative hypothesis and your conclusion.

Solution:

$$H_0: \mu = 385$$

$$H_1: \mu \neq 385$$

Method 1: (Using a confidence interval)

As 385 is in the confidence interval above,  $[361.53, 386.47]$ , we fail to reject  $H_0$ .

Method 2: (Using the test statistic  $Z$ )

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{374 - 385}{\frac{45}{\sqrt{50}}} = -1.73$$

As,  $-1.96 < -1.73 < 1.96$ , we fail to reject  $H_0$ .